A Statistical Theory of Chord under Churn [*]

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Abstract. *Most earlier studies of DHTs under churn have either depended on simulations as the primary investigation tool, or on establishing bounds for DHTs to function. In this paper, we present a complete analytical study of churn using a master-equation-based approach, used traditionally in non-equilibrium statistical mechanics to describe steady-state or transient phenomena. Simulations are used to verify all theoretical predictions. We demonstrate the application of our methodology to the Chord system. For any rate of churn and stabilization rates, and any system size, we accurately predict the fraction of failed or incorrect successor and finger pointers and show how we can use these quantities to predict the performance and consistency of lookups under churn. We also discuss briefly how churn may actually be of different 'types' and the implications this will have for the functioning of DHTs in general.*

1 Introduction

Theoretical studies of asymptotic performance bounds of DHTs under churn have been conducted in works like [6, 2]. However, within these bounds, performance can vary substantially as a function of different design decisions and configuration parameters. Hence simulation-based studies such as [5, 8, 3] often provide more realistic insights into the performance of DHTs. Relying on an understanding based on simulations alone is however not satisfactory either, since in this case, the DHT is treated as a black box and is only empirically evaluated, under certain operation conditions. In this paper we present an alternative theoretical approach to analyzing and understanding DHTs, which aims for an accurate prediction of performance, rather than on placing asymptotic performance bounds. Simulations are then used to verify all theoretical predictions.

Our approach is based on constructing and working with master equations, a widely used tool wherever the mathematical theory of stochastic processes is applied to real-world phenomena [7]. We demonstrate the applicability of this approach to one specific DHT: Chord [9]. For Chord, it is natural to define the state of the system as the state of all its nodes, where the state of an alive node is specified by the states of all its pointers. These pointers (either fingers or successors) are then in one of three states: alive and correct, alive and incorrect or failed. A master equation for this system is simply an equation for the time evolution of the probability that the system is in a particular state. Writing such an equation involves keeping track of all the gain/loss terms which add/detract from this probability, given the details of the dynamics. This approach is applicable to any P2P system (or indeed any system with a discrete set of states).

Our main result is that, for every outgoing pointer of a Chord node, we systematically compute the probability that it is in any one of the three possible states, by computing all the gain and loss terms that arise from the details of the Chord protocol under churn. This probability is different for each of the successor and finger pointers. We then use this information to predict both lookup consistency (number of failed lookups) as well as lookup performance (latency) as a function of the parameters involved. All our results are verified by simulations.

The main novelty of our analysis is that it is carried out entirely from first principles *i.e.* all quantities are predicted solely as a function of the parameters of the problem: the churn rate, the stabilization rate and the number of nodes in the system. It thus differs from earlier related theoretical studies where quantities similar to those we predict, were either assumed to be *given* [10], or *measured* numerically [1].

Closest in spirit to our work is the informal derivation in the original Chord paper [9] of the average number of timeouts encountered by a lookup. This quantity was approximated there by the product of the average number of fingers used in a lookup times the probability that a given finger points to a departed node. Our methodology not only allows us to derive the latter quantity rigorously but also demonstrates how this probability depends on which finger (or successor) is involved. Further we are able to derive an exact relation relating this probability to lookup performance and consistency accurately at any value of the system parameters.

2 Assumptions & Definitions

Basic Notation. In what follows, we assume that the reader is familiar with Chord. However we introduce the notation used below. We use $\mathcal{K}$ to mean the size of the Chord key space and N the number of nodes. Let $\mathcal{M} = \log_2 \mathcal{K}$ be the number of fingers of a node and $\mathcal{S}$ the length of the immediate successor list, usually set to a value $= O(\log(N))$. We refer to nodes by their

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keys, so a node n implies a node with key $n \in 0 \cdots \mathcal{K}-1$. We use p to refer to the predecessor, s for referring to the successor list as a whole, and s_i for the i^{th} successor. Data structures of different nodes are distinguished by prefixing them with a node key e.g. $n'.s_1$, etc. Let fin_i.*start* denote the start of the i^{th} finger (Where for a node n, $\forall i \in 1..\mathcal{M}$, $n.fin_i.start = n + 2^{i-1}$) and fin_i.*node* denote the actual node pointed to by that finger.

Steady State Assumption. λ_j is the rate of joins per node, λ_f the rate of failures per node and λ_s the rate of stabilizations per node. We carry out our analysis for the general case when the rate of doing successor stabilizations $\alpha\lambda_s$, is not necessarily the same as the rate at which finger stabilizations $(1-\alpha)\lambda_s$ are performed. In all that follows, we impose the steady state condition $\lambda_j = \lambda_f$. Further it is useful to define $r \equiv \frac{\lambda_s}{\lambda_f}$ which is the relevant ratio on which all the quantities we are interested in will depend, e.g, $r = 50$ means that a join/fail event takes place every half an hour for a stabilization which takes place once every 36 seconds.

Parameters. The parameters of the problem are hence: $\mathcal{K}$, N, α and r. All relevant measurable quantities should be entirely expressible in terms of these parameters.

Chord Simulation. We use our own discrete event simulation environment implemented in Java which can be retrieved from [4]. We assume the familiarity of the reader with Chord, however an exact analysis necessitates the provision of a few details. Successor stabilizations performed by a node n on $n.s_1$ accomplish two main goals: i) Retrieving the predecessor and successor list of of $n.s_1$ and reconciling with n's state. ii) Informing $n.s_1$ that n is alive/newly joined. A finger stabilization picks one finger at random and looks up its start. Lookups do not use the optimization of checking the successor list before using the fingers. However, the successor list is used as a last resort if fingers could not provide progress. Lookups are assumed not to change the state of a node. For joins, a new node u finds its successor v through some initial random contact and performs successor stabilization on that successor. All fingers of u that have v as an acceptable finger node are set to v. The rest of the fingers are computed as best estimates from $v's$ routing table. All failures are ungraceful. We make the simplifying assumption that communication delays due to a limited number of hops is much smaller than the average time interval between joins, failures or stabilization events. However, we do not expect that the results will change much even if this were not satisfied.

Averaging. Since we are collecting statistics like the probability of a particular finger pointer to be wrong, we need to repeat each experiment 100 times before obtaining well-averaged results. The total simulation sequential real time for obtaining the results of this paper was about 1800 hours that was parallelized on a cluster of 14 nodes where we had $N = 1000$, $\mathcal{K} = 2^{20}$, $\mathcal{S} = 6$, $200 \le r \le 2000$ and $0.25 \le \alpha \le 0.75$.

3 The Analysis

3.1 Distribution of Inter-Node Distances

During churn, the inter-node distance (the difference between the keys of two consecutive nodes) is a fluctuating variable. An important quantity used throughout the analysis is the pdf of inter-node distances. We define this quantity below and state a theorem giving its functional form. We then mention three properties of this distribution which are needed in the ensuing analysis. Due to space limitations, we omit the proof of this theorem and the properties here and provide them in [4].

Definition 3.1 *Let $Int(x)$ be the number of intervals of length x, i.e. the number of pairs of consecutive nodes which are separated by a distance of x keys on the ring.*

Theorem 3.1 *For a process in which nodes join or leave with equal rates (and the number of nodes in the network is almost constant) independently of each other and uniformly on the ring, The probability ($P(x) \equiv \frac{Int(x)}{N}$) of finding an interval of length x is:*

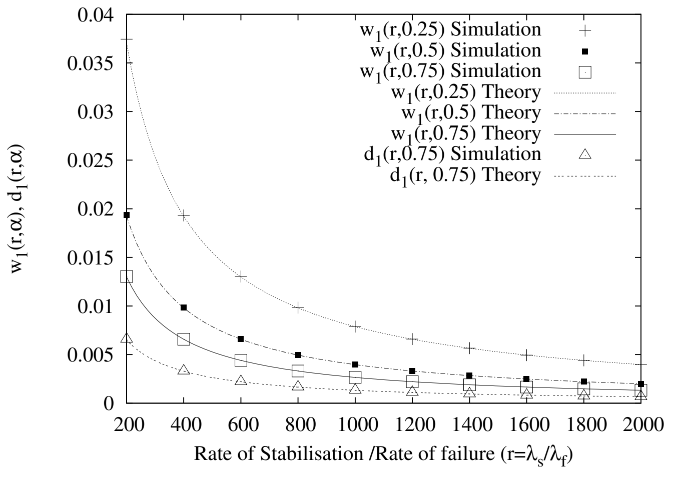
$$P(x) = \rho^{x-1}(1-\rho) \text{ where } \rho = \frac{\mathcal{K}-N}{\mathcal{K}} \text{ and } 1-\rho = \frac{N}{\mathcal{K}}$$

The derivation of the distribution $P(x)$ is independent of any details of the Chord implementation and depends solely on the join and leave process. It is hence applicable to any DHT that deploys a ring.

Property 3.1 *For any two keys u and v, where $v = u + x$, let b_i be the probability that the first node encountered inbetween these two keys is at $u + i$ (where $0 \le i < x - 1$). Then $b_i \equiv \rho^i(1-\rho)$. The probability that there is definitely atleast one node between u and v is: $a(x) \equiv 1 - \rho^x$. Hence the conditional probability that the first node is at a distance i given that there is atleast one node in the interval is $bc(i,x) \equiv b(i)/a(x)$.*

Property 3.2 *The probability that a node and atleast one of its immediate predecessors share the same k^{th} finger is $p_1(k) \equiv \frac{\rho}{1+\rho}(1-\rho^{2^k-2})$. This is $\sim 1/2$ for $\mathcal{K} >> 1$ and $N << \mathcal{K}$.Clearly $p_1 = 0$ for $k = 1$. It is straightforward (though tedious) to derive similar expressions for $p_2(k)$ the probability that a node and atleast two of its immediate predecessors share the same k^{th} finger, $p_3(k)$ and so on.*

Property 3.3 *We can similarly assess the probability that the join protocol (see previous section) results in further replication of the k^{th} pointer. That is, the probability that a newly joined node will choose the k^{th} entry of its successor's finger table as its own k^{th} entry is $p_{\text{join}}(k) \sim \rho(1-\rho^{2^{k-2}-2}) + (1-\rho)(1-\rho^{2^{k-2}-2}) - (1-\rho)\rho(2^{k-2}-2)\rho^{2^{k-2}-3}$. The function $p_{\text{join}}(k) = 0$ for small k and 1 for large k.*



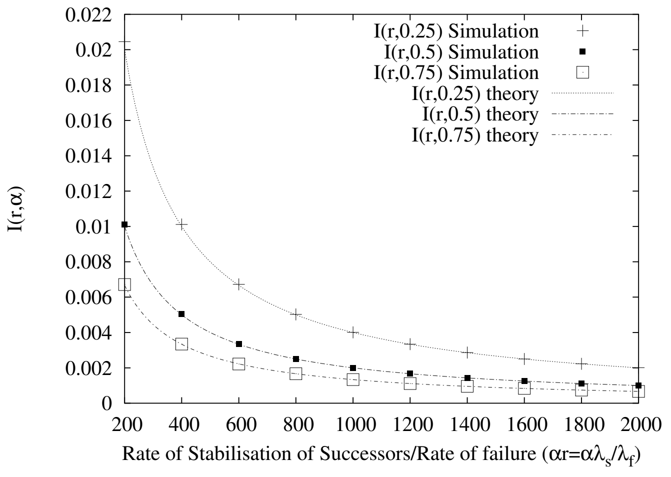


Figure 1: Theory and Simulation for $w_1(r,\alpha)$, $d_1(r,\alpha)$, $I(r,\alpha)$

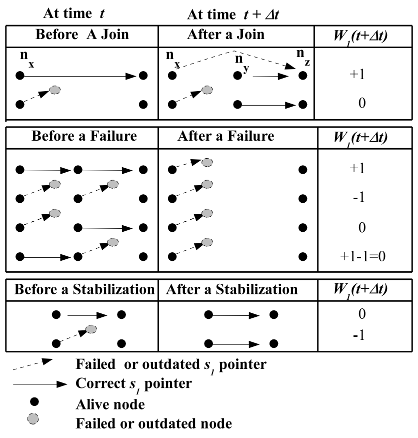


Figure 2: Changes in W_1, the number of wrong (failed or outdated) s_1 pointers, due to joins, failures and stabilizations.

3.2 Successor Pointers

In order to get a master-equation description which keeps all the details of the system and is still tractable, we make the ansatz that the state of the system is the product of the states of its nodes, which in turn is the product of the states of all its pointers. As we will see this ansatz works very well. Now we need only consider how many kinds of pointers there are in the system and the states these can be in. Consider first the successor pointers.

Let $w_k(r,\alpha)$, $d_k(r,\alpha)$ denote the fraction of nodes having a *wrong* k^{th} successor pointer or a *failed* one respectively and $W_k(r,\alpha)$, $D_k(r,\alpha)$ be the respective *numbers* . A *failed* pointer is one which points to a departed node and a *wrong* pointer points either to an incorrect node (alive but not correct) or a dead one. As we will see, both these quantities play a role in predicting lookup consistency and lookup length.

By the protocol for stabilizing successors in Chord, a node periodically contacts its first successor, possibly correcting it and reconciling with its successor list. Therefore, the number of wrong k^{th} successor pointers are not independent quantities but depend on the number of wrong first successor pointers. We consider only s_1 here.

Change in $W_1(r,\alpha)$	Rate of Change
$W_1(t+\Delta t) = W_1(t)+1$	$c_1 = (\lambda_j \Delta t)(1-w_1)$
$W_1(t+\Delta t) = W_1(t)+1$	$c_2 = \lambda_f(1-w_1)^2\Delta t$
$W_1(t+\Delta t) = W_1(t)-1$	$c_3 = \lambda_f w_1^2 \Delta t$
$W_1(t+\Delta t) = W_1(t)-1$	$c_4 = \alpha\lambda_s w_1 \Delta t$
$W_1(t+\Delta t) = W_1(t)$	$1-(c_1+c_2+c_3+c_4)$

Table 1: Gain and loss terms for $W_1(r,\alpha)$: the number of wrong first successors as a function of r and α.

We write an equation for $W_1(r,\alpha)$ by accounting for all the events that can change it in a micro event of time Δt. An illustration of the different cases in which changes in W_1 take place due to joins, failures and stabilizations is provided in figure 2. In some cases W_1 increases/decreases while in others it stays unchanged. For each increase/decrease, table 1 provides the corresponding probability.

By our implementation of the join protocol, a new node n_y, joining between two nodes n_x and n_z, has its s_1 pointer always correct after the join. However the state of $n_x.s_1$ before the join makes a difference. If $n_x.s_1$ was correct (pointing to n_z) before the join, then after the join it will be wrong and therefore W_1 increases by 1. If $n_x.s_1$ was wrong before the join, then it will remain wrong after the join and W_1 is unaffected. Thus, we need to account for the former case only. The probability that $n_x.s_1$ is correct is $1-w_1$ and from that follows the term c_1.

For failures, we have 4 cases. To illustrate them we use nodes n_x, n_y, n_z and assume that n_y is going to fail. First, if both $n_x.s_1$ and $n_y.s_1$ were correct, then the failure of n_y will make $n_x.s_1$ wrong and hence W_1 increases by 1. Second, if $n_x.s_1$ and $n_y.s_1$ were both wrong, then the failure of n_y will decrease W_1 by one, since one wrong pointer disappears. Third, if $n_x.s_1$ was wrong and $n_y.s_1$ was correct, then W_1 is unaffected. Fourth, if $n_x.s_1$ was correct and $n_y.s_1$ was wrong, then the wrong pointer of n_y disappeared and $n_x.s_1$ became wrong, therefore W_1 is unaffected. For the first case to happen, we need to pick two nodes with correct pointers, the probability of this is $(1-w_1)^2$. For the second case to happen, we need to pick two nodes with wrong pointers, the probability of this is w_1^2. From these probabilities follow the terms c_2 and c_3.

Finally, a successor stabilization does not affect W_1, unless the stabilizing node had a wrong pointer. The probability of picking such a node is w_1. From this follows the term c_4.

Hence the equation for $W_1(r,\alpha)$ is:

$$\frac{dW_1}{dt} = \lambda_j(1-w_1) + \lambda_f(1-w_1)^2 - \lambda_f w_1^2 - \alpha\lambda_s w_1$$

Solving for w_1 in the steady state and putting $\lambda_j = \lambda_f$, we get:

$$w_1(r,\alpha) = \frac{2}{3+r\alpha} \approx \frac{2}{r\alpha} \quad (1)$$

This expression matches well with the simulation results as shown in figure 1. $d_1(r,\alpha)$ is then $\approx \frac{1}{2}w_1(r,\alpha)$ since when $\lambda_j = \lambda_f$, about half the number of wrong pointers are incorrect and about half point to dead nodes. Thus $d_1(r,\alpha) \approx \frac{1}{r\alpha}$ which also matches well the simulations as shown in figure 1. We can also use the above reasoning to iteratively get $w_k(r,\alpha)$ for any k.

Lookup Consistency By the lookup protocol, a lookup is inconsistent if the immediate predecessor of the sought key has an wrong s_1 pointer. However, we need only consider the case when the s_1 pointer is pointing to an alive (but incorrect) node since our implementation of the protocol always requires the lookup to return an alive node as an answer to the query. The probability that a lookup is inconsistent $I(r,\alpha)$ is hence $w_1(r,\alpha) - d_1(r,\alpha)$. This prediction matches the simulation results very well, as shown in figure 1.

3.3 Failure of Fingers

We now turn to estimating the fraction of finger pointers which point to failed nodes. As we will see this is an important quantity for predicting lookups. Unlike members of the successor list, alive fingers even if outdated, always bring a query closer to the destination and do not affect consistency. Therefore we consider fingers in only two states, alive or dead (failed).

Let $f_k(r,\alpha)$ denote the fraction of nodes having their k^{th} finger pointing to a failed node and $F_k(r,\alpha)$ denote the respective

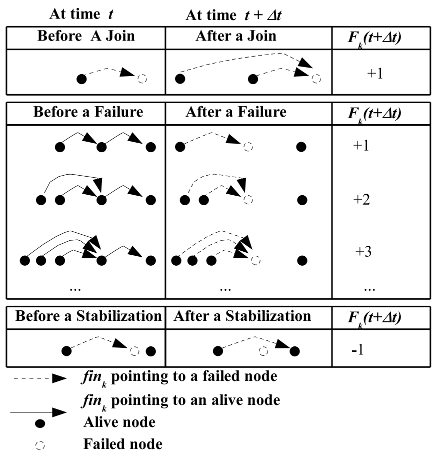


Figure 4: Changes in F_k, the number of failed fin_k pointers, due to joins, failures and stabilizations.

$F_k(t+\Delta t)$	Rate of Change
$= F_k(t)+1$	$c_1 = (\lambda_j\Delta t)p_{join}(k)f_k$
$= F_k(t)-1$	$c_2 = (1-\alpha)\frac{1}{\mathcal{M}}f_k(\lambda_s\Delta t)$
$= F_k(t)+1$	$c_3 = (1-f_k)^2[1-p_1(k)](\lambda_f\Delta t)$
$= F_k(t)+2$	$c_4 = (1-f_k)^2(p_1(k)-p_2(k))(\lambda_f\Delta t)$
$= F_k(t)+3$	$c_5 = (1-f_k)^2(p_2(k)-p_3(k))(\lambda_f\Delta t)$
$= F_k(t)$	$1-(c_1+c_2+c_3+c_4+c_5)$

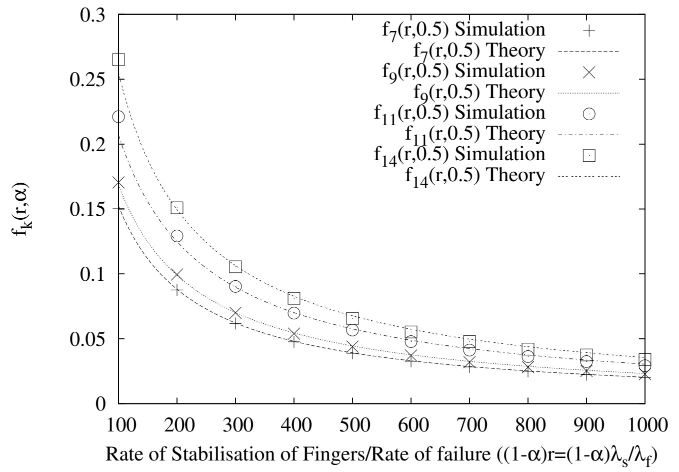
Table 2: Some of the relevant gain and loss terms for F_k, the number of nodes whose kth fingers are pointing to a failed node for $k > 1$.

number. For notational simplicity, we write these as simply F_k and f_k. We can predict this function for any k by again estimating the gain and loss terms for this quantity, caused by a join, failure or stabilization event, and keeping only the most relevant terms. These are listed in table 2.

A join event can play a role here by increasing the number of F_k pointers if the successor of the joinee had a failed k^{th} pointer (occurs with probability f_k) and the joinee replicated this from the successor (occurs with probability $p_{join}(k)$ from property 3.3).

A stabilization evicts a failed pointer if there was one to begin with. The stabilization rate is divided by $\mathcal{M}$, since a node stabilizes any one finger randomly, every time it decides to stabilize a finger at rate $(1-\alpha)\lambda_s$.

Given a node n with an alive k^{th} finger (occurs with probability $1-f_k$), when the node pointed to by that finger fails, the number of failed k^{th} fingers (F_k) increases. The amount of this increase depends on the number of immediate predecessors of n that were pointing to the failed node with their k^{th} finger. That number of predecessors could be 0, 1, 2,.. etc. Using property 3.2 the respective probabilities of those cases are: $1-p_1(k)$, $p_1(k)-p_2(k)$, $p_2(k)-p_3(k)$,... etc.



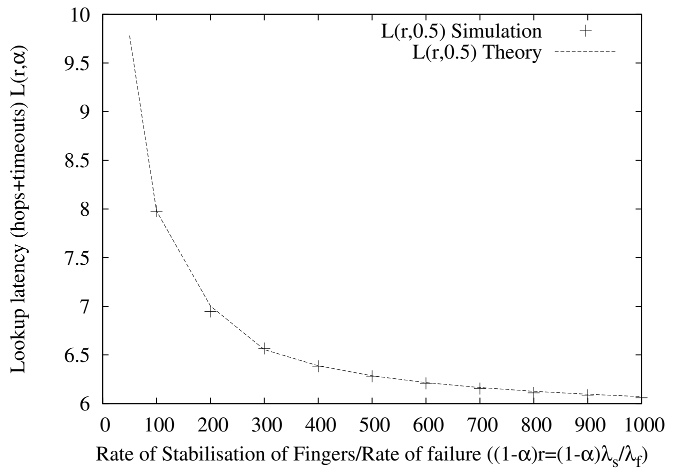


Figure 3: Theory and Simulation for $f_k(r,\alpha)$, and $L(r,\alpha)$

Solving for f_k in the steady state, we get:

$$f_k = \frac{\left[2\tilde{P}_{rep}(k) + 2 - p_{\text{join}}(k) + \frac{r(1-\alpha)}{\mathcal{M}}\right]}{2(1+\tilde{P}_{rep}(k))} - \frac{\sqrt{\left[2\tilde{P}_{rep}(k) + 2 - p_{\text{join}}(k) + \frac{r(1-\alpha)}{\mathcal{M}}\right]^2 - 4(1+\tilde{P}_{rep}(k))^2}}{2(1+\tilde{P}_{rep}(k))} \quad (2)$$

where $\tilde{P}_{rep}(k) = \Sigma p_i(k)$. In principle its enough to keep even three terms in the sum. The above expressions match very well with the simulation results (figure 3).

3.4 Cost of Finger Stabilizations and Lookups

In this section, we demonstrate how the information about the failed fingers and successors can be used to predict the cost of stabilizations, lookups or in general the cost for reaching any key in the id space. By cost we mean the number of hops needed to reach the destination *including* the number of timeouts encountered en-route. For this analysis, we consider timeouts and hops to add equally to the cost. We can easily generalize this analysis to investigate the case when a timeout costs some factor n times the cost of a hop.

Define $C_t(r,\alpha)$ (also denoted C_t) to be the expected cost for a given node to reach some target key which is t keys away from it (which means reaching the first successor of this key). For example, C_1 would then be the cost of looking up the adjacent key (1 key away). Since the adjacent key is always stored at the first alive successor, therefore if the first successor is alive (occurs with probability $1-d_1$), the cost will be 1 hop. If the first successor is dead but the second is alive (occurs with probability $d_1(1-d_2)$), the cost will be 1 hop + 1 timeout = 2 and the *expected* cost is $2 \times d_1(1-d_2)$ and so forth. Therefore, we have $C_1 = 1 - d_1 + 2 \times d_1(1-d_2) + 3 \times d_1 d_2(1-d_3) + \cdots \approx 1 + d_1 = 1 + 1/(\alpha r)$.

For finding the expected cost of reaching a general distance t we need to follow closely the Chord protocol, which would lookup t by first finding the closest preceding finger. For notational simplicity, let us define ξ to be the start of the finger (say the k^{th}) that most closely precedes t. Thus $t = \xi + m$, i.e. there are m keys between the sought target t and the start of the most closely preceding finger. With that, we can write a recursion relation for $C_{\xi+m}$ as follows:

$$\begin{aligned} C_{\xi+m} &= C_\xi\left[1-a(m)\right] \\ &+ (1-f_k)\left[a(m) + \sum_{i=1}^{m} b_{m+1-i} C_i\right] \\ &+ f_k a(m)\Bigg[1 + \sum_{i=1}^{k-1} h_k(i) \\ &\sum_{l=1}^{\xi/2^i} bc(l,\xi/2^i)(1 + C_{\xi_i+1-l+m}) + 2h_k(k)\Bigg] \end{aligned} \quad (3)$$

where $\xi_i \equiv \sum_{m=1,i} \xi/2^m$ and $h_k(i)$ is the probability that a node is forced to use its $k-i^{th}$ finger owing to the death of its k^{th} finger. The probabilities a, b, bc have already been introduced in section 3.

The lookup equation though rather complicated at first sight merely accounts for all the possibilities that a Chord lookup will encounter, and deals with them exactly as the protocol dictates. The first term accounts for the eventuality that there is no node intervening between ξ and $\xi + m$ (occurs with probability $1 - a(m)$). In this case, the cost of looking for $\xi + m$ is the same as the cost for looking for ξ. The second term accounts for the situation when a node does intervene inbetween (with probability $a(m)$), and this node is alive (with probability $1 - f_k$). Then the query is passed on to this node (with 1 added to register the increase in the number of hops) and then the cost depends on the length of the distance between this node and t. The third term accounts for the case when the intervening node is dead (with probability f_k). Then the cost increases by 1 (for a timeout) and the query needs to be passed back to the closest preceding finger. We hence compute the probability $h_k(i)$ that it is passed back to the $k - i^{th}$ finger either because the intervening fingers are dead or share the same finger table entry as the k^{th} finger. The cost of the lookup now depends on the remaining distance to the sought key. The expression for $h_k(i)$ is easy to compute using theorem 3.1 and the expression for the f_k's [4].

The cost for general lookups is hence

$$L(r, \alpha) = \frac{\Sigma_{i=1}^{\mathcal{K}-1} C_i(r, \alpha)}{\mathcal{K}}$$

The lookup equation is solved recursively, given the coefficients and C_1. We plot the result in Fig 3. The theoretical result matches the simulation very well.

4 Discussion and Conclusion

We now discuss a broader issue, connected with churn, which arises naturally in the context of our analysis. As we mentioned earlier, all our analysis is performed in the steady state where the rate of joins is the same as the rate of departures. However this rate itself can be chosen in different ways. While we expect the mean behaviour to be the same in all these cases, the fluctuations are very different with consequent implications for the functioning of DHTs. The case where fluctuations play the least role are when the join rate is "per-network" (The *number* of joinees does not depend on the current number of nodes in the network) and the failure rate is "per-node" (the number of failures does depend on the current number of occupied nodes). In this case, the steady state condition is $\lambda_j / N = \lambda_f$ guaranteeing that N can not deviate too much from the steady state value. In the two other cases where the join and failure rate are both per-network or (as in the case considered in this paper) both per-node, there is no such "repair" mechanism, and a large fluctuation can (and will) drive the number of nodes to extinction, causing the DHT to die. In the former case, the time-to-die scales with the number of nodes as $\sim N^3$ while in the latter case it scales as $\sim N^2$ [4]. Which of these 'types' of churn is the most relevant? We imagine that this depends on the application and it is hence probably of importance to study all of them in detail.

To summarize, in this paper, we have presented a detailed theoretical analysis of a DHT-based P2P system, Chord, using a Master-equation formalism. This analysis differs from existing theoretical work done on DHTs in that it aims not at establishing bounds, but on precise determination of the relevant quantities in this dynamically evolving system. From the match of our theory and the simulations, it can be seen that we can predict with an accuracy of greater than 1% in most cases.

Apart from the usefulness of this approach for its own sake, we can also gain some new insights into the system from it. For example, we see that the fraction of dead finger pointers f_k is an increasing function of the length of the finger. Infact for large enough $\mathcal{K}$, all the long fingers will be dead most of the time, making routing very inefficient. This implies that we need to consider a different stabilization scheme for the fingers (such as, perhaps, stabilizing the longer fingers more often than the smaller ones), in order that the DHT continues to function at high churn rates. We also expect that we can use this analysis to understand and analyze other DHTs.

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